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MAINTENANCE AND REPLACEMENT MODELS

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CHAPTER – I

PART - A

1.1 MAINTENANCE

Maintenance is a routine activity of keeping an equipment or a machine or a facility in its normal working condition so that it can perform its intended function satisfactorily without causing any loss to the service time on account of its breakdown .

Maintenance is any activity designed to keep the resources in good working condition or restore them to operating status. The significant resource used to add value to products is the equipment or machinery. Therefore, the machinery should be maintained in good working state. If not, there may be too much down time and break for production, if the machinery is a part of an assembly line. Improper functioning of equipment may lead to issues related to quality. So, it would be good if the equipment or machinery are always maintained in working condition with minimum possible cost. Therefore, minimization of overall maintenance cost needs a wholistic approach. Sometimes the machinery may be outdated over a time horizon. If an organization or company wishes to continue competitive business, it is essential to look at whether to maintain with the existing (old) machinery or to replace considering the costs of operations and maintenance.

Maintenance activities pertaining to facilities and equipment in good working condition are indispensable to achieve specified level of quality, reliability and efficient working. Maintenance activity helps in maintaining and increasing the operational efficiency of equipment or facilities and, thus, contributes to revenue by reducing the operating costs and increasing the effectiveness of production.

As soon as an equipment is designed, manufactured and setup, the maintenance crew will take care of its operational availability. The thought of taking up maintenance activity is of ages; it was introduced as early as an equipment was. Decades ago, the equipment or machinery were used till its failure and then it would be repaired if possible, or else discarded.

In order to maximize the availability, the sophisticated equipment must be maintained properly throughout the life. Improved automation and the need for safety made the maintenance managers to develop appropriate maintenance techniques for different machinery.

1.2 NEED FOR MAINTENANCE

"*Maintenance is a function to keep the equipment or machinery in their normal operating condition by servicing or repairing or replacing some of the components*". The concept of maintenance is to prepare a plan of action regarding the steps to be followed while carrying out a particular maintenance activity. Based on history of equipment and the feedback obtained from users, concrete procedures for maintenance are developed and together called "maintenance policies", the development of which requires manufacturer-user interaction. As a result, the manufacturer may redesign the equipment as maintenance requirements of the user. The basic need for maintenance of any equipment or machinery is to:

1. Minimize the frequency of interruptions to production by reducing breakdowns.
2. Maximize the production capacity from the given equipment resources.
3. Enhance the safety of work force.
4. Enhance the service reliability of equipment.

5. Enable to achieve the quality of a product or service through well-maintained equipment.
6. Maximize the useful life of the equipment.
7. Reduce the repair or breakdown cost component in the total production cost.

1.3 OBJECTIVES OF MAINTENANCE

In order to accomplish the goals of the organization, the objectives of maintenance must be set within the framework of the organizational structure. Therefore, it is essential for the maintenance section to make sure that:

a) Equipments are in good working condition at lowest running cost possible.
b) Delivery schedules are not changed because of unavailability of equipment in good operating condition.
c) The equipments' performance is reliable.
d) Equipment is maintained to minimum breakdowns.
e) Equipment's life is extended.

As the activity of maintenance adds some more cost to the running cost besides improving the performance of the equipment and availability in optimum working condition, "establishing a balance between the availability and overall running costs should be objective of maintenance work". Therefore, ensuring the availability of equipments for maximum time at minimal cost is the responsibility of the maintenance function. Hence, departmentation or creation of various sections is done in industries, now-a-days, to take care of the maintenance activities of equipments.

1.4 CHALLENGES IN MAINTENANCE

Challenges faced by maintenance function of the modern industry are attributable to:

- The current technology is becoming obsolete with hasty growth of technology. Such a challenge is a frequent one in Mechatronics Engineering industry where in electronics and mechanical technologies are embedded in one system. For e.g. Air Conditioning systems.
- Evolution of latest analytical tools, hasty repair systems.
- Need of balancing the machines (both old and latest machines) in service as several industrial organizations possessing the blend of old equipment using outdated technology and modern equipment using latest technology.

Effective management of maintenance function in exigent environment is sometimes a critical situation. In addition to the solving the problems with the equipment, the maintenance section also takes care of:

- Modernization of the equipment, machinery
- Imbibing the required technical skills in the maintenance crew
- Effective and efficient use of both the maintenance crew and the old equipment to attain better availability.
- Looking for optimality in all maintenance related activities
- Overhauling of old spares.
- Effective utilization of the maintenance personnel
- Establishing in house research and development center to bring development in maintenance methods.

1.5 TYPES OF MAINTENANCE

Martand Telsang (1998) [8] classified the maintenance activity into two types: i) Planned Maintenance ii) Unplanned Maintenance

Various types of maintenance and their relationships are represented in figure 1.1.

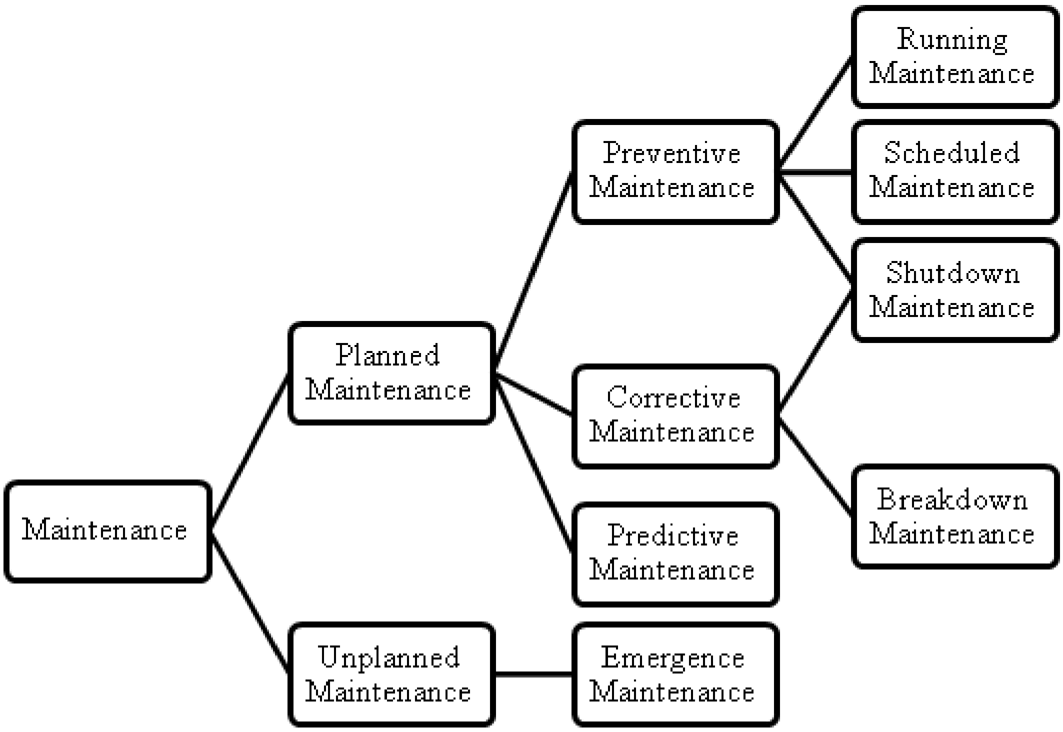

Figure 1.1 Types of Maintenance

1.5.1 PLANNED MAINTENANCE (PM)

This is an organized type of maintenance work carried out as per recorded procedures having control. To evade breakdowns, the maintenance tasks are preplanned considering ‘when and what kind of the maintenance works’, and ‘who would take up the maintenance work’. To meet the requirements of the planned maintenance, ‘Work Study’ has to be carried out to decide the cycle time of maintenance and conducting the ‘Time Study’ may also be helpful and suggestive for developing economical maintenance schedules for the equipments.

In this maintenance, directives and commands are to be greater in detail and equipment specific. The condition of the equipment must be monitored regularly, where safety is of vital importance. Following is the sub-classification of Planned Maintenance (PM).

- Preventive Maintenance
    - Scheduled Maintenance
- Corrective Maintenance
    - Breakdown Maintenance
- Predictive Maintenance

1.5.1.1 PREVENTIVE MAINTENANCE

The preventive maintenance policy is a system of planned and scheduled maintenance with the aim of minimizing or preventing breakdowns. Preventive maintenance is "*the utilization of planned and coordinated inspections, repairs, adjustments, and replacements in maintaining an equipment or plant*". The premise behind the preventive maintenance is that "*prevention is better than cure*". Preventive Maintenance (PM) is "*the periodical inspection and service activities, which are aimed to detect potential failures and perform minor adjustments or repairs, which will prevent major operating problems in future*". One of the main objectives of preventive maintenance is to detect any condition that may cause failure of the machine before such breakdown occurs. This system makes it possible to plan and schedule the maintenance tasks with no disturbance in production schedule and hence improves the equipment availability. All the components or at least the critical elements of the equipment will be systematically inspected at predetermined time intervals to disclose the situation that may lead to production break and unsafe depreciation. In general preventive maintenance plan is not readily available to outfit any industry. To make it appropriate to the needs of an organization it must be custom-made. Preventive maintenance includes Failure analysis and planning for their elimination.

As preventive maintenance replaces the weak components identified during the inspection, its planning and execution is expensive. Preventive maintenance is suitable where the occurrence of operational risk because of equipment's failure must be avoided. However, maintenance costs get compensated in terms of extended life of equipment. By and large, this system is put into action in complex plants to evade grave breakdowns. Scheduled Maintenance is one type of preventive maintenance.

SCHEDULED MAINTENANCE:

In this system, maintenance schedule is prepared consulting the manufacturing division so that particular machine / part of the process can be undertaken for maintenance task. By deciding the maintenance frequency in advance with experience, the equipment's idle time, the maintenance work force can be effectively utilized and also the experts will be made readily available for executing maintenance. The scheduled maintenance improves the equipment's availability, even though it appears to be costly compared to breakdown maintenance. Scheduled maintenance is generally used to overhaul the equipment. There are two types of preventive maintenance schedules.

i) Firm Schedule or Fixed Time Maintenance Schedule

Most of preventive maintenance schedules are of firm schedules. Workload cycles will be scheduled for different time durations like: daily, weekly, biweekly, monthly, bimonthly, quarterly, half-yearly, or yearly etc., Crew carrying out preventive maintenance jobs should normally be separate from crew attending breakdowns. During the equipment's lifetime, the frequency of schedule will not be same. Failure rates are high during initial stage that is just after commissioning, and are low during useful period / normal working period and once again high towards the end of the working cycle that is just before the discard of equipment or before major over hauling.

ii) Condition Based Maintenance (CBM)

In this method, the condition of the equipment or some critical parts of the equipment are continuously monitored using sophisticated monitoring instruments so that the failure may be predicted well before it occurs and corrective steps are taken to prevent failure.

Predictive maintenance is more feasible today because of technology that available for equipment surveillance and diagnosis of problems while the machines are still operating. The condition of a machine can be monitored by several means. Sensors may be installed, or periodic readings may be taken with portable units to measure vibration or temperature. Vibration sensors and ultrasonic sensors are used to feed data in to a computer for analysis. Tie deviation from the normal vibration pattern are recorded when the machine is running properly are analyzed to determine where the problem is developing and when it will become serious. A problem of this type prevents unplanned downtime that disrupts production schedules.

The objectives of condition-based maintenance are:

- To detect the failures before they occur.
- To carry out maintenance only when required.
- To reduce the maintenance costs and downtime costs.

The methodology of Condition Based Maintenance

1. Proper identification and location of machines/equipments by codification.
2. Selection of critical machines and systems.
3. Identifying components/elements, which are failure prone.
4. Fixing condition parameters.
5. Monitoring techniques.
6. Monitoring schedule and frequency.
7. Trend monitoring.
8. Repair schedule and execution.
9. Follow up.

Benefits of condition monitoring

- Increased system availability
- Improved plant operation and safety
- Improved maintenance
- Improved product quality

1.5.1.2 CORRECTIVE MAINTENANCE

It is one type of planned maintenance and can be defined as "*the practice carried out to restore the full performance of the equipment that has stopped working to acceptable standards. For instance, an engine may be in working condition, but does not make its full load because of worn–out piston rings. Thus, if the piston rings are replaced, it will bring back the performance of the engine to specified level*". Corrective Maintenance, if properly implemented will result in reduction of maintenance costs and equipments' downtime as well.

Restoration of failed units is the primary intention of corrective maintenance, which is a one stretch job that must be fully completed once the activity is engaged. Corrective maintenance accentuate in getting complete information of every breakdown and the reason for each. Efforts are made to identify and eliminate the cause by activities such as improving maintenance practices, changing frequency of maintenance services and improving process control procedures.

The use of planned preventive maintenance gives out a clear picture regarding the recurring failures of a particular component of an equipment such that the recurrence can be avoided and also informed to the manufacturer to incorporate changes in the design of equipments.

1.5.2 BREAKDOWN MAINTENANCE

The Corrective maintenance can be further classified into:

- Breakdown Maintenance
- Shutdown Maintenance

Breakdown Maintenance:

Breakdown maintenance is an emergency based policy in which the plant or equipment is operated until it fails and then it is brought back into running condition by repair. The maintenance staff locate any mechanical, electrical or any other fault to correct it immediately.

The beak down maintenance policy may work good and feasible for small factories where:

a) there are few types of equipment
b) machines/equipments are simple and does not require any specialist
c) sudden failure does not cause any serious financial loss.

The basic idea of breakdown maintenance is not to do anything when everything goes right. Thus, there would not be any maintenance work as long as the equipment functioning is satisfactory. As soon as equipment fails from normal functioning, repair work would be taken up to restore the equipment to normal operating condition. This type of maintenance is economical for such equipment involving less downtime and repair cost as well.

Drawbacks of Breakdown Maintenance:

- Equipment's operational availability will be less.
- Components for repair or replacement might be unavailable sometimes with maintenance stores.
- Major loss of productive time of both machines and the work men, if there is any delay in restoring the equipments to working condition.
- Focus is on repairing and restoring the equipment to working condition but not on knowing the reason for failure
- Danger of recurrence of failures

This type of maintenance is economical for that equipment, whose breakdown time and repair costs are less. However, there are several limitations with Breakdown maintained in case of high cost production systems.

1.6 PREVENTIVE VERSUS BREAKDOWN MAINTENANCE

Preventive maintenance is the routine inspection and service activities designed to detect potential failure conditions and make major adjustments or repairs that will help prevent major operating problems whereas, breakdown maintenance is the emergency repair and it involves higher cost of facilities and equipments that have been used until they fail to operate.

An effective preventive maintenance programme for equipments require properly trained personnel, regular inspection and service and should maintain regular records. It is planned in such a way that it will not disturb the normal operations, hence, no down time cost of equipment. Breakdown maintenance stops the normal activities and the machines as well as the operators are rendered idle till the equipment is brought back to normal condition of working.

Breakdown maintenance is the repair that is generally done after the equipment has attained down state. It is often of an emergency nature that will have associated penalty in terms of expediting cost of maintenance and down time cost of equipments. Preventive maintenance will reduce such cost up to a point. Beyond that time, the cost of preventive maintenance will be more when compared to the down time cost. Under such situation, a firm can opt for break-down maintenance. These concepts are shown in figure 1.2.

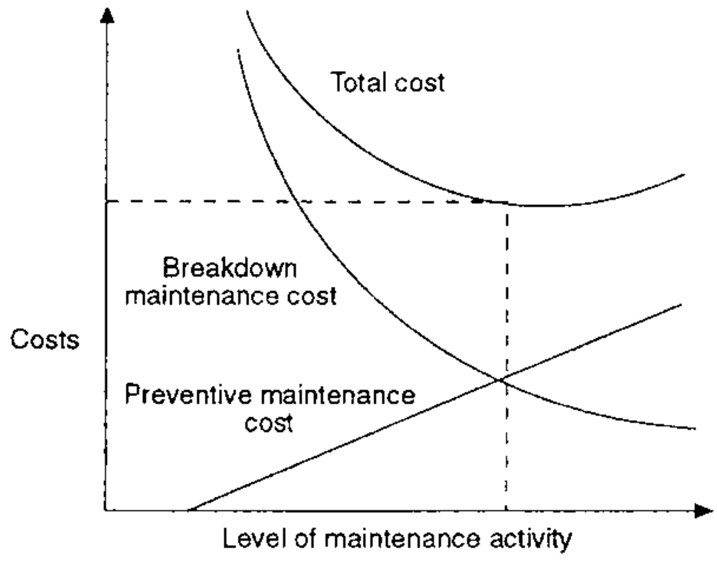

Figure 1.2 Preventive Vs. Breakdown Maintenance Costs

1.7 BENEFITS OF MAINTENANCE

A huge investment of capital in any manufacturing or service sectors anticipates suitable returns through the equipment. Such anticipations would become true, when the equipment delivers normal functioning. Often the maintenance schedules given by the equipment supplier do not convey the desired output and equipment's life. Therefore, sometimes it would be necessary for proper maintenance of the equipment to achieve the required levels of product or service outputs. When the system of maintenance is well organized one, the benefits to the organization are:

(a) Minimization of Equipments' down time

(b) Increase availability of the entire system

(c) Enhancement of equipments' productive life

(d) Safe working environment to the workmen.

When the equipment is a part of a production system, there would be a great loss because of its failure as it causes the entire production system to shut down. Adopting appropriate maintenance schedule will reduce the rate of wear in the equipment also. Sometimes, the workers' safety may be most important and in such cases the appropriate preventive maintenance would assure them the safety. For instance, every aircraft must be inspected prior and later to air travel keeping in view the safety of the passengers.

1.8 MAINTENANCE COSTS

Equipment's breakdown results in loss of production, costly emergency repairs, delays in production schedules besides keeping the men and machinery idle. As shown in figure 1.3, the costs of break down generally surpass total cost of preventive maintenance, which includes like cost of inspection, cost of service and scheduled repairs up to the point 'M'. Beyond this optimal point, an increasingly higher level of preventive maintenance is not economically justified and it is economical to adopt breakdown maintenance policy. The optimal level of maintenance activity M is easily identified on a theoretical basis, to do this the details of the costs associated with breakdown and preventive maintenance must be known. Various cost components of maintenance include:

1. Downtime (Idle time cost) cost due to equipment breakdown.
2. Cost of spares or other material used for repairs
3. Cost of maintenance labour and overheads of maintenance departments.
4. Losses due to inefficient operations of machines.
5. Capital money required for equipments' replacement.

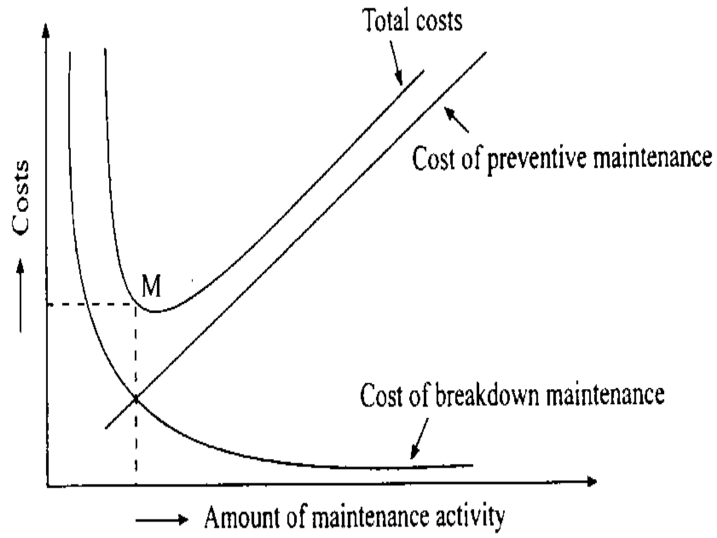

Figure 1.3 Costs associated with Maintenance activity

1.9 MAINTENANCE PLANNING AND SCHEDULING

Once the list of defects is known and the type of the maintenance is decided, the next step is to plan and schedule the maintenance jobs in order to execute the jobs properly and to get the desired results. Planning and scheduling of maintenance jobs/activities are the important components of maintenance function.

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PART – B

1.10 THE REPLACEMENT PROBLEM

The replacement problems deal with deal with the situation that arise when some components (men or machinery) requires replacement because of reduced efficiency, or breakdown or complete failure. Such decreased efficiency or complete failure may be either gradual or all of a sudden.

A model is a replica of a real or existing system. It demonstrates the inter-relationships between various decision variables in the system. Providing a path to analyze the system behavior for improving its performance should be the primary objective of a model. The mathematical model uses a set of equations, establishing a relation between the decision variables, to describe the system behavior.

The need for decision of replacement is raised in any organization both in case of men and machinery. From the theory of probability, it is possible to guess the chance of failure at various stages.

1.11 OBJECTIVES OF REPLACEMENT

The primary objective of replacement is to direct the organization towards profit maximization or cost minimization. Deciding the replacement policy that determines the optimal replacement age of equipment, instead of using with higher maintenance costs for long time, is the main objective of replacement problem. For instance, in order to replace an:

- item whether to wait till its failure or replacing at an early age with higher cost.
- equipment whether to replace the inefficient equipment with a similar type of equipment or with a modern one.

In case of high cost equipment, the decision is whether to replace it immediately or later.

1.12 FAILURE MECHANISMS OF EQUIPMENTS

The term 'failure' has a wider meaning in *business* than what it has in our daily life. Failures can be discussed under two categories viz., Gradual Failures, and Sudden Failures.

1.12.1 GRADUAL FAILURE

The mechanism under this category is progressive. That is, as the life of an item increases, its efficiency deteriorates, causing:

- Increased expenditure for operating costs
- Decreased equipments' productivity
- Decrease in the value of the equipment

Example: bearings, pistons, piston rings, 'Automobile Tyres', mechanical systems like machines, machine tools, flexible manufacturing equipment etc. fall under this category.

1.12.2 SUDDEN FAILURE

This type of failure is applicable to those items that do not deteriorate markedly with service, but which ultimately fail after some period of using. For any particular type of equipment the period from installation to failure is not equal but will follow some 'frequency distribution which may be progressive, retrogressive, or random in nature'.

a) **Progressive failures:** In this mechanism, probability of failure increases as the life of equipment increases. Examples include: e*lectric light bulbs*, *automobile tubes* etc.,

b) **Retrogressive failures:** Some equipment may prone to failure with high probability in the beginning of their life, and as the time progresses the probability of failure falls down. i.e., the capability of the equipment to survive in the beginning of life enhances its probable life. Industrial equipments with this type of distribution of life span is exemplified by aircraft engines.

c) **Random failures:** Under this failure, constant probability of failure is associated with the equipment that fails from random causes such as physical shocks, not related to age. In such a case, virtually all equipments fail prior to their expected life. Example: Electronic components like transistors, semi conductor elements, glass made items, delicate or brittle items, perishable items like fruits and vegetables' have been shown to fail at a rate independent of the age.

1.13 NEED FOR REPLACEMENT

Replacement becomes necessary when the job-performing units such as men, machines, equipment, parts etc. lose their efficiency and effectiveness because of gradual deterioration or sudden failure or breakdown. Planned replacement of these items would reduce maintenance cost and other overhead expenses. When a machine loses its efficiency gradually the maintenance becomes very expensive. Therefore, the problem is to determine the age at which it is most economical to replace the item. On the other hand, certain items such as bulbs, radio, television, and computer parts fail suddenly without giving any indication of failure and they become completely useless. These items are to be replaced immediately as and when they fail to function.

The two basic reasons for considering the replacement of an equipment or a machine are physical impairment and obsolescence.

Physical impairment refers only to changes in the physical condition of the machine itself. This would lead to a decline in the value of the service rendered, increased operating cost, increased maintenance cost or a combination of these.

Obsolescence is mainly due to improvement of the tools of production, improvement in technology.

Therefore, it would be uneconomical to continue production with the same machine under any of the above situations. Hence, the machines are to be periodically replaced.

The replacement situation arises due to the following reasons.

(1) Weak performance of the existing equipment and needs expensive maintenance.

(2) Failure of the existing equipment because of industrial accident or some other reason, or anticipating the failure of an existing equipment soon.

(3) Availability of mechanized or fully automated modern equipment with better design, made the existing equipment outdated.

The equipment, whose efficiency gradually decreases according to their age, requires paying out more money towards running cost, and scrap etc., Therefore, the only alternative way to prevent such increased expenses is the replacement of old equipment with new one.

Replacement problems fall into the following categories depending upon the life pattern of the equipment involved.

- Replacement of the equipment that wears out or becomes obsolete with time (because of constant use or new technological developments)

- Replacement of the equipment that fails completely.
    - Individual Replacement Policy : Mortality Theorem
    - Group Replacement of items that fail completely
- Other replacement problems
    - Recruitment and Promotion Problems
    - Equipment Renewal Problems
- Miscellaneous problems

'For items that wear out, the problem is to balance the cost of new equipment against the cost of maintaining the efficiency on the old and / or cost due to the loss of efficiency. Though, general solution is not possible, models have been constructed and solutions have been derived using simplified assumptions about the conditions of the problem.

A separate but similar problem involves the replacement of items such as electric bulb, radio tubes, etc. of some electronic equipment, which does not deteriorate with time but suddenly fails. The problem, in this case, is of finding which items to replace and whether or not to replace them in a group and, if so, when. The objective is to minimize the sum of the cost of the item, cost of replacing the item and the cost associated with failure of the item'.

Sharma S.D (2004) [6], "Operations Research", Kedarnath Ramnath & Co Publishers, Meerut has given various replacement models as follows.

1.14 REPLACEMENT OF ITEMS WHICH DETERIORATE AND WHOSE MAINTENANCE COST INCREASES WITH TIME

Replacement of items that deteriorate with time will depend upon the value of money and the corresponding associated costs.

Costs to be considered: Various costs that are to be included in this model are all those costs that depend upon the choice or age of the equipment. In some special cases, certain costs need to be included in the calculations. For example, in considering the optimum decisions of replacement for a particular machine, the costs that do not change with the age of the equipment need not be considered. The costs to be considered while calculating the optimum replacement period are:

(i) Capital Recovery Cost =Average First Cost, if rate of interest is zero percent.

(ii) Running cost = Average operating and maintenance cost (O&M cost).

The above associated costs can be expressed as average cost per period and the sum of the above two costs can be considered as total cost.

Economic Life of Equipment / Justification of Replacement

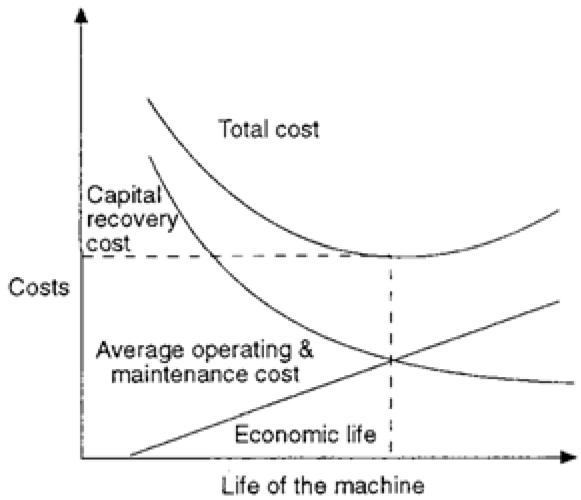

Figure 1.4 Chart Showing Economic Life

A typical shape of each of the above costs with respect to life of the machine is shown in figure 1.4, from which it is clear that the capital recovery cost (average first cost) goes on decreasing with the life of the equipment and the average operating and maintenance cost goes on increasing with the life of the equipment. From the beginning, the total cost goes on decreasing up to a particular life and then it starts increasing. The economic life of the equipment is the point corresponding to the minimum total cost.

It can be observed that the average cost per period go on decreasing, longer the replacement is postponed. However, there comes an age at which the average cost per period tend to increase. Thus, at this age the replacement is justified.

A machine loses efficiency with time and we have to determine the best time at which we have to go for a new one. In case of a vehicle, the maintenance cost is increasing as it is getting aged. These costs increase day by day if we postpone the replacement.

If the interest rate is more than zero percent, then we use interest formulas to determine the economic life. The replacement alternatives can be evaluated based on present worth criterion and annual equivalent criterion.

1.14.1 Replacement of Items Whose Maintenance and Repair Cost Increases With Time, Ignoring the Changes in the Value of Money During the Period

THEOREM: *The cost of maintenance of a machine is given as a function increasing with time and its scrap value is constant.*

a) *"If time is measured continuously, then the average annual cost will be minimized by replacing the machine when the average cost today becomes equal to the current maintenance cost".*

b) *"If time is measured in discrete units, then the average annual cost will be minimized by replacing the machine when the next period's maintenance cost becomes greater than the current average cost".*

<u>PROOF:</u> (a) 'When time 't' is a continuous variable'

Let, C = Capital cost of the item,

R_t = Operating and Maintenance cost of the item at time 't',

S = Scrap value of the item,

n = number of years the item is to be in use,

$T(n)$ = Total Cost incurred during 'n' years

T_{avg} = Average annual cost of the item,

Obviously, annual cost of the item at any time 't' = $C - S + R_t$.

Since, the maintenance cost incurred during 'n' years becomes, = $\int_0^n R_t \, dt$,

The total cost incurred on the item will become: $T(n) = C - S + \int_0^n R_t \, dt$

Hence, average annual cost is given by, $T_{avg} = \dfrac{T(n)}{n} = \dfrac{1}{n}\left(C - S + \int_0^n R_t \, dt \right)$ ---(1.1)

Now, we shall find the value of 'n' for which T_{avg} is minimum. Therefore, differentiating T_{avg} with respect to 'n',

$$\frac{d}{dn}(T_{avg}) = -\frac{1}{n^2}(C - S) - \frac{1}{n^2}\int_0^n R_t \, dt + \frac{1}{n}R_t \qquad \text{---(1.2)}$$

For minimum of T_{avg}, $\dfrac{d}{dn}(T_{avg}) = 0$

We have, $R_n = \dfrac{1}{n}(C - S) + \dfrac{1}{n}\int_0^n R_t \, dt = T_{avg}$, ---(1.3)

Thus, the item should be replaced when the average annual cost to date becomes equal to the current maintenance cost.

(b) 'When time 't' is a discrete variable'

In this case, the total cost incurred during 'n' years, $T(n) = C - S + \sum_{t=1}^{n} R_t$ and the average annual cost is given by, $T_{avg} = \frac{T(n)}{n} = \frac{1}{n}\left(C - S + \sum_{t=1}^{n} R_t \right)$ ---(1.4)

The average total cost T_{avg} will be minimum, provided the following relationship is satisfied: $\Delta T_{avg}(n-1) < 0 < \Delta T_{avg}(n)$ (By using finite differences) ---(1.5)

Differentiating equation (1.4), 'under the summation sign by definition of first difference', $\Delta T_{avg} = T_{avg}(n+1) - T_{avg}(n)$

$$= \left[\sum_{t=1}^{n+1} \frac{R_t}{n+1} + \frac{C-S}{n+1} \right] - \left[\sum_{t=1}^{n} \frac{R_t}{n} + \frac{C-S}{n} \right]$$

$$= \left[\frac{R_{n+1}}{n+1} + \sum_{t=1}^{n} \frac{R_t}{n+1} \right] - \left[\sum_{t=1}^{n} \frac{R_t}{n} \right] + (C-S)\left[\frac{1}{n+1} - \frac{1}{n} \right]$$

$$= \frac{R_{n+1}}{n+1} + \sum_{t=1}^{n} R_t \left[\frac{1}{n+1} - \frac{1}{n} \right] + (C-S)\left[\frac{1}{n+1} - \frac{1}{n} \right]$$

… (1.15a)

$$= \frac{R_{n+1}}{n+1} - \sum_{t=1}^{n} \frac{R_t}{n(n+1)} - \frac{(C-S)}{n(n+1)}$$

Since $T_{avg} > 0$, for minimum of T_{avg}. So,

$$\frac{R_{n+1}}{n+1} > \sum_{t=1}^{n} \frac{R_t}{n(n+1)} + \frac{(C-S)}{n(n+1)} \qquad \text{or} \qquad R_{n+1} > \sum_{t=1}^{n} \frac{R_t}{n} + \frac{(C-S)}{n}$$

or $R_{n+1} > \frac{T(n)}{n}$, by virtue of equation (1.4) …(1.5b)

Similarly, it can be shown that : $R_n < \frac{T(n)}{n}$, by virtue of $\Delta T_{avg}(n-1) < 0$.

That is 'n' is optimal when the average annual cost is minimum.

Therefore, $R_{n+1} > \frac{T(n)}{n} > R_n$.

Thus, the theorem is proved.

1.14.2 MONEY VALUE, PRESENT WORTH FACTOR (PWF) AND DISCOUNT RATE

Time Value of Money: Generally, in establishing mathematical models for optimum replacements, always the value of money does not remain constant and it is subjected to change over a period of time and so the value of money play an important role in model development.

Conceptually, 'time value of money' means that the value of a unit of money is different in different time periods. The value of a sum of money received today is more than its value received after some time. Conversely, the sum of money received in future is less valuable than it is today. In other words, the present worth of a rupee received after some time will be less than a rupee received today. Since a rupee received today has more value, rational investors would prefer current receipt to future receipts. The time value of money can also be referred to as **'time preference for money'.**

PRESENT WORTH FACTOR (PWF):

The value of money over a period of time, depends upon the nominal interest rate 'r'. The value of one rupee today would be equal to $Rs.1\left(1+\text{r}\%\right)$ after one year. Or the present value of a rupee to be spent after one year is equal to $Rs.1(1+r\%)^{-1}$ at the interest rate r % per year. Similarly, the present value of a rupee to be spent after 'n' years is equal to $\left(1+\text{r}\right)^{-\text{n}}$, and is called as '**Present Worth Factor (PWF)'** or **'Present Value Interest Factor, PVIF (r%, n)** at the rate of r % per 'n' years. Sometimes, this is also known as '**Compound Amount Factor (CAF)'** of one rupee spent in 'n' year duration.

1.14.3 Replacement of Items Whose Maintenance and Repair Cost Increases With Time, Value of Money also Changes With Time

As already explained in the preceding section, the money value can be interpreted in two different ways. Accordingly, the optimal replacement policy can be determined by the following two methods:

a) "The maintenance cost increases with time, and the money value decreases with constant rate i.e., depreciation value is given".

b) "The amount to be spent is borrowed at a given rate of interest under the condition of repaying it in pre-decided number of installments".

Theorem: '*The maintenance cost increases with time and money value decreases at constant rate i.e., depreciation value is given, then the replacement policy will be'*:

i) 'Replace if the next period's maintenance cost is greater than the weighted average cost of previous periods'.

ii) 'Do not replace if the next period's cost is less than the weighted average cost of the previous periods'.

PROOF: (METHOD-1)

Suppose that the item (equipment or a machine) is available for use over a series of time periods of equal intervals (say, one year),

Let us assume,

C = 'Capital Cost or Purchase Price of the item to be replaced'

R_i = 'Running cost or maintenance cost incurred at the beginning of the i^{th} year'

r = rate of interest

$v=(1+r)^{-1}$ = the present worth of a rupee to be spent a year hence.

The proof can be divided into two major steps:

Step-1: 'To find the Present Worth of total expenditure'

Let, the item is considered for replacement at the end of every n[th] year. The year wise present worth of costs on the item in the successive cycles of 'n' years can be computed as shown in Table 1.1.

Table 1.1 Present worth (Year wise) of future costs

Year	1	2	…	n	n+1	n+2	…	2n	2n+1
Present Worth	$C+R_1$	$R_2 v$	…	$R_n v^{n-1}$	$(C+R_1)v^n$	$R_2 v^{n+1}$	…	$R_2 v^{2n-1}$	$(C+R_1)v^{2n}$

Assuming that the item has no resale price at the time of replacement, the present worth of all future discounted costs associated with the policy of replacing the item at the end of every 'n' years will be given by ,

$$T(n) = [(C+R_1) + R_2 v + \ldots + R_n v^{n-1}] + [(C+R_1)v^n + R_2 v^{n+1} + \ldots + R_n v^{2n-1}] +$$

$$[(C+R_1)v^{2n} + R_2 v^{2n+1} + \ldots + R_n v^{3n-1}] + \ldots \text{and so on.}$$

Summing up the right hand side, we get ,

$$T(n) = (C+R_1)(1+v^n+v^{2n}+\ldots) + R_2 v(1+v^n+v^{2n}+\ldots) + \ldots + R_n v^{n-1}(1+v^n+v^{2n}+\ldots)$$

$$= \left(C+R_1+R_2 v+\cdots+R_n v^{n-1}\right)\left(1+v^n+v^{2n}+\cdots\right)$$

$$= \left(C+R_1+R_2 v+\cdots+R_n v^{n-1}\right)\frac{1}{1-v^n} \quad \text{--- (1.6)}$$

$\left(\text{Since, } v<1, \text{the sum of infinite Geometric Progression is } \frac{1}{1-v^n}\right)$

$$T(n) = \frac{T_{avg}(n)}{1-v^n},\ T(n+1) = \frac{T_{avg}(n+1)}{1-v^{n+1}}. \quad \text{---(1.7)}$$

where, for simplicity, $T_{avg} = C+R_1 + R_2 v + \ldots + R_n v^{n-1}]$

Step-2: 'To determine replacement policy so that T(n) is minimum'.

Since n is measure in discrete units, we shall use the method of finite difference in order to minimize the present worth expenditure T(n).
Thus, $T(n)$ will be minimum, if $T(n+1) > T(n-1), \textit{i.e.}, \Delta T(n) > 0 > \Delta T(n-1)$.

So by the definition of first difference,

$$\Delta T(n) = T(n+1) - T(n) = \frac{T_{avg}(n+1)}{1-\nu^{n+1}} - \frac{T_{avg}(n)}{1-\nu^{n}}, \quad \text{from equation (1.7)}$$

$$= \frac{(1-\nu^{n})\,T_{avg}(n+1) - (1-\nu^{n+1})\,T_{avg}(n)}{(1-\nu^{n+1})(1-\nu^{n})},$$

First, it is convenient to simplify the numerator of $\Delta T(n)$ only. That is

$$N^{r} = (1-\nu^{n})\,T_{avg}(n+1) - (1-\nu^{n+1})\,T_{avg}(n)$$

$$= [T_{avg}(n+1) - T_{avg}(n)] + \nu^{n+1}\,T_{avg}(n) - \nu^{n} T_{avg}(n+1)$$

$$= [R_{n+1}\nu^{n} + \nu^{n+1}\,T_{avg}(n) - \nu^{n}[T_{avg}(n) + \nu^{n} R_{n+1}]$$

$$= R_{n+1}\nu^{n} + \nu^{n+1}\,T_{avg}(n) - \nu^{n}[T_{avg}(n) + \nu^{n} R_{n+1}] \qquad (\because T_{avg}(n+1) = T_{avg}(n) + \nu^{n} R_{n+1})$$

$$\therefore \Delta T(n) = \frac{\nu^{n}(1-\nu^{n})R_{n+1} - \nu^{n}(1-\nu)T_{avg}(n)}{(1-\nu^{n+1})(1-\nu^{n})}$$

$$\Delta T(n) = \frac{\nu^{n}(1-\nu)}{(1-\nu^{n+1})(1-\nu^{n})}\left[\frac{(1-\nu^{n})}{(1-\nu)} R_{n+1} - T_{avg}(n)\right] \qquad \text{---(1.8)}$$

Simply setting (n – 1) for 'n' in (1.8)

$$\Delta T(n-1) = \frac{\nu^{n-1}(1-\nu)}{(1-\nu^{n})(1-\nu^{n-1})}\left[\frac{(1-\nu^{n-1})}{(1-\nu)} R_{n} - T_{avg}(n-1)\right]$$

After little simplifications on RHS,

$$\Delta T(n-1) = \frac{\nu^{n-1}(1-\nu)}{(1-\nu^{n})(1-\nu^{n-1})}\left[\frac{(1-\nu^{n})}{(1-\nu)} R_{n} - T_{avg}(n)\right] \qquad \text{---(1.9)}$$

$$\because \left[\frac{(1-\nu^{n-1})}{(1-\nu)}R_n - T_{avg}(n-1)\right] = \left[\frac{(1-\nu^{n-1})}{(1-\nu)}R_n - \left(T_{avg}(n) - R_n\nu^{n-1}\right)\right]$$

$$= \left[\frac{(1-\nu^{n-1})}{(1-\nu)} + \nu^{n-1}\right]R_n - T_{avg}(n) = \left(\frac{(1-\nu^{n})}{(1-\nu)}R_n - T_{avg}(n)\right)$$

The quantity $\frac{\nu^n(1-\nu)}{(1-\nu^{n+1})(1-\nu^n)}$ in equation (1.8) is greater than zero at all the times, $\because |\nu| < 1.$

Thus, $\Delta T(n)$ has the same sign as the quantity in (1.8), is with similar explanation for $\Delta T(n-1)$ in (1.9) also. Therefore, for present worth expenditure to become minimum, the condition, $\Delta T(n-1) < 0 < \Delta T(n)$, becomes

$$\left(\frac{(1-\nu^{n})}{(1-\nu)}R_n - T_{avg}(n)\right) < 0 < \left(\frac{(1-\nu^{n})}{(1-\nu)}R_{n+1} - T_{avg}(n)\right) \quad \text{---(1.10)}$$

$$\frac{(1-\nu^{n})}{(1-\nu)}R_n < T_{avg}(n) < \frac{(1-\nu^{n})}{(1-\nu)}R_{n+1} \quad \text{---(1.11)}$$

$$R_n < \frac{C + R_1 + R_2 v + \cdots + R_n v^{n-1}}{1 + v + v^2 + \cdots + v^{n-1}} < R_{n+1} \quad \text{---(1.12)}$$

$$R_n < \frac{T_{avg}(n)}{\sum \nu^{n-1}} < R_{n+1} \quad \text{---(1.13)}$$

The expression between R_n and R_{n+1} in (1.12) and (1.13) is called the 'weighted average cost' of previous 'n' years with weights $1, v, v^2, \cdots, v^{n-1}$ respectively.

The value of 'n' satisfying the relationship (1.10) or (1.12) will be the best replacement age of the item. This proves the theorem.

<u>Special Case:</u>

It is interesting to note that the replacement policy given by equation (1.5b), (when money value was not counted) is a limiting case of that given by equation

(1.12) (when money value is considered). Because, if the interest rate r→0, then v→1. Thus, taking limit of (1.12) as v→1, we get,

$$R_{n+1} > \frac{C+R_1+R_2+R_3+....+R_n}{(1+1+1+...+1)\,(n\ \text{times})} > R_n$$

$$\text{i.e., } R_{n+1} > \frac{T(n)}{n} > R_n$$

(When scrap value S is zero) which is identical to the result (1.5b)

PROOF: (METHOD-2)

Under the assumptions stated in first method, we suppose that the amount equal to the present worth expenditure T(n) is taken as loan (borrowed) at the rate of 'r %' per year and this amount T(n) is repaid through fixed annual payments during the life time of the equipment. Therefore, the variable payment actually made are converted into fixed annual payments 'x'. Hence, 'the present worth of fixed annual payments (x) for each cycle of 'n' years becomes equal to the sum borrowed'. That is,

if $\nu = \dfrac{1}{(1+r)}$, we have T(n)

$$= [x + \nu x + ... + \nu^{n-1}x] + [\nu^n x + \nu^{n+1}x + ... + \nu^{2n-1}x] + [\nu^{2n}x + \nu^{2n+1}x + ... + \nu^{3n-1}x] + ..$$

$$= x(1+\nu+....+\nu^{n-1}) + \nu^n x(1+\nu+....+\nu^{n-1}) + \nu^{2n}x(1+\nu+....+\nu^{n-1}) + ...$$

$$= (1+\nu+....+\nu^{n-1})(x+\nu^{2n}x+....)$$

$$= \left(\frac{1-\nu^n}{1-\nu}\right)\left(\frac{x}{1-\nu^n}\right) = \left(\frac{x}{1-\nu}\right) \quad \text{or} \quad x = (1-\nu)\,T(n) \qquad \textbf{---(1.14)}$$

Since, $(1-\nu)$ is positive constant, 'x' will be minimum when T(n) is minimum. The condition for T(n) being minimum has already been obtained in step-2 of method-1.

It is interesting to observe from equations (1.7) and (1.14), that

$$x=(1-\nu)T(n)=(1-\nu)\frac{T_{avg}(n)}{(1-\nu^n)}=\frac{T_{avg}(n)}{(1-\nu^n)/(1-\nu)}, \quad \text{or} \quad x=\frac{C+R_1+R_2\nu+.....+R_n\nu^{n-1}}{(1+\nu+....+\nu^{n-1})}$$

$$x=\frac{T_{avg}(n)}{\sum \nu^{n-1}} \qquad \left(Since,\ 1+\nu+....+\nu^{n-1}=\frac{1-\nu^n}{1-\nu}\right) \qquad \textbf{---(1.15)}$$

∴ Annual payment (x) = Weighted average cost for ‘n’ years.

Now proceed to determine a policy for selecting an economically best item from amongst those available from various factories.

1.15 REPLACEMENT OF ITEMS THAT FAIL COMPLETELY AND SUDDENLY

A system generally consists of a huge number of low-priced components that are increasingly liable to failure with age. Electronic items like bulbs, resistors, tube lights etc., generally fail all of a sudden, instead of a gradual deterioration. The sudden failure of the item results in complete breakdown of the system. The system may contain a collection of such items or just an item like a single tube light. The costs of failure, in such a case will be fairly more than the cost of the item itself. In addition, the value of the failed item is so small that the cost of keeping records of individual ages cannot be justified. For example, a tube or a condenser in an aircraft costs little, but the failure of such a low cost item may lead the airplane to crash. Hence we use some replacement policy for such items which would minimize the possibility of complete breakdown. The following are the replacement policies, which are applicable for this situation.

(i) ***Individual replacement policy*** in which an item is replaced immediately after it fails.

(ii) ***Group replacement policy*** is concerned with those items that either work or fail completely. In this policy, a decision is made as regard to ‘at what equal intervals, all the items are to be replaced simultaneously irrespective of whether they have failed or not, with a provision to replace the items individually, which fail during the fixed group replacement period'.

There is a trade-off between the individual replacement policy and the group replacement policy. Hence, for a given problem, each of the replacement policies is evaluated and the most economical policy is selected for implementation. The optimal period of replacement is determined by calculating the minimum total cost. The total cost is calculated using: probability of failure at time 't', number of items failing during time 't', cost of group replacement and the cost of individual replacement.

1.15.1 INDIVIDUAL REPLACEMENT POLICY (MORTALITY THEOREM)

In this policy, 'an equipment or item is to be replaced as soon as it fails. Mortality tables will be referred to determine the life span of any equipment or the probability distribution of failure. To discuss such type of replacement policy, the problem of human population is considered. No group of people ever existed under the conditions.

a) that all deaths are immediately replaced by births, and
b) that there are no other entries or exits.

Nevertheless, the reason for stating the problem under these two assumptions is that the analysis becomes more easy by keeping the virtual human population in mind. Later, such problems can also be translated into industrial items, where death is equivalent to a part failure and birth is equivalent to new replacement. Thus, industries also face a fairly common situation. The following ***Mortality Theorem*** will make the conceptions clear'.

MORTALITY THEOREM

"*A large population is subject to a given mortality law for a very long period of time. All deaths are immediately replaced by births and there are no other entries or exits. Then the age distribution ultimately becomes stable and that the number of deaths per unit time (failure of industrial items per unit time) becomes constant (which is equal to the size of the total population divided by the mean age at death)*".

PROOF:

For the sake of convenience, let us assume that 'each death occurs just before some time t=w, where 'w' is an integer and that no member of the population remains alive longer than w+1 time units'.

Let, f(t) = no. of births at time 't',

p(x) = probability of member dying just before age x+1, i.e. at the age 'x'.

since, f(t-x) represents the number of births at time (t-x), the age of such members who remain alive at time 't' will obviously be 'x'. This can be understood from the figure 1.5.

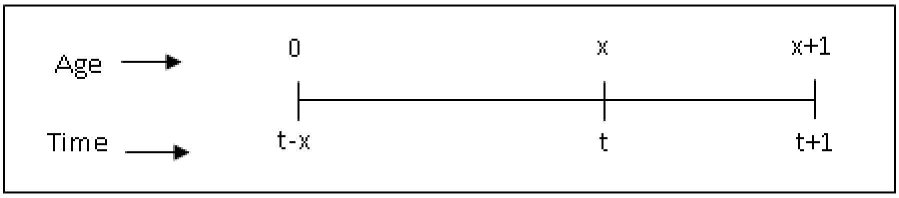

Figure 1.5 Relation between age and time period

So, their former probability of dying at time 't' (just before time 't+1') will be equal to the probability p(x) of those dying at age 'x' (just before age x+1). Hence, the expected number of deaths of such alive members at time 't' is p(x).f(t−x).

Therefore, the total number of deaths at time 't' will be

$$= \sum_{x=0}^{w} f(t-x).p(x), \quad t = w, w+1, w+2, \ldots\ldots \qquad ---(1.16)$$

Also, total number of births at time (t+1) = f(t+1).

Since all deaths at time 't' are immediately replaced by births at time (t+1), therefore

$$f(t+1)=\sum_{x=0}^{w} f(t-x)p(x), \qquad \text{---(1.17)}$$

The differential equation (1.17) in 't', may be solved by substituting f(t)=Aα^t

Then the differential equation (1.17) becomes, $A\alpha^{t+1} = A\sum_{x=0}^{w} \alpha^{t-x} p(x)$

On dividing by $A\alpha^{t-w}$, we get $\alpha^{w+1} = \sum_{x=0}^{w} \alpha^{w-x} p(x)$ *Or* $\alpha^{w+1} - \sum_{x=0}^{w} \alpha^{w-x} p(x) = 0$

Or $\alpha^{w+1} - \left[\alpha^{w} p(0) + \alpha^{w-1} p(1) + \alpha^{w-2} p(2) + \cdots + p(w)\right] = 0$ --- (1.18)

∵ Sum of all probabilities is equal to one, i.e., $\sum_{x=0}^{w} p(x) = 1$ **or** $1 - \sum_{x=0}^{w} p(x) = 0$

or $\sum_{x=0}^{w} p(x) = 1 - \left[p(0) + p(1) + p(2) + ... + p(w)\right] = 0$ …(1.19)

'Now comparing equations (1.18) and (1.19) it is found that one solution of (1.18) is $\alpha_0 = 1$. However, the polynomial equation (1.18) must have (w+1) total number of roots. Let, the remaining roots be denoted by $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_w$, consequently, the solution of difference equation (1.17) will be of the form':

$$f(t) = A_0 + A_1\alpha_1^t + A_2\alpha_2^t + \ldots + A_w\alpha_w^t \qquad \text{---(1.20)}$$

'Where $A_0, A_1, \ldots, A_w$ are constants, whose value can be determined with the help of age distribution at some given point in time. Further, it can be shown that the absolute value of all the remaining roots is less than unity i.e., $|\alpha_i| < 1$, for $i = 1, 2, 3, \ldots, w$.

Hence, $\alpha_1^t, \alpha_2^t, \ldots, \alpha_w^t$ tends to zero as $t \to \infty$. Consequently, equation (1.20) becomes $f(t) = A_0$, which shows that the number of deaths per unit time (as well as the number of births) becomes constant'.

‘To show that the age distribution ultimately becomes stable’:

Let, the probability of survivor for more than ‘x’ time units be P(x), then,

$$P(x) = 1 - \{p(0) + p(1) + p(2) + ... + p(x\text{-}1)\} \text{ and } P(0) = 1 \quad \text{---(1.21)}$$

‘Since the number of births as well as deaths have become constant, each equal to A_0, the expected number of survivors of age ‘x’ is given by $A_0P(x)$. As the deaths are immediately replace by births, the size N of population remains constant. That is

$$N = A_0 \sum_{x=0}^{w} P(x) \quad \textbf{or} \quad \text{---(1.22)}$$

$$A_0 = \frac{N}{\sum_{x=0}^{w} P(x)} \quad \text{---(1.23)}$$

Now, ‘the number of survivors aged 0,1,2,3,…… can be computed from equation (1.22) as A_0, $A_0P(1)$, $A_0P(2)$, ... so on ’.

Finally, ‘if the denominator in (1.23) became the mean age at death, then the age distribution will become stable’. For this,

$$\sum_{x=0}^{w} P(x) = \sum_{x=0}^{w} P(x)\ \Delta(x) \quad [\because \Delta(x) = (x+1) - x \text{ by finite differences}]$$

$$\sum_{x=0}^{w} P(x) = [P(x).x]_0^{w+1} - \sum_{x=0}^{w} (x+1)\ \Delta P(x)$$

$$\sum_{x=0}^{w} P(x) = [P(w+1).(w+1) - 0] - \sum_{x=0}^{w} (x+1)\ \Delta P(x) \quad \text{---(1.24)}$$

But, $P(w+1) = 1 - p(0) - p(1) - p(2) ...... - p(w)$ [$\because$ equation (1.21)]

$= 1 - 1 = 0.$ [$\because$ eqn.(1.19)]

Since, no one can survive for more than (x+1) units of age, and

$$\Delta P(x) = P(x+1) - P(x)$$

$$= [1 - p(0) - p(1) - p(2) ...... - p(x)] - [1 - p(0) - p(1) - p(2) ...... - p(x\text{-}1)] = -p(x)$$

Therefore, 'substituting the simplified values of P(w+1) and ΔP(x) in equation (1.24)'

to obtain, $\sum_{x=0}^{w} P(x) = 0 + \sum_{x=0}^{w} (x+1)\ P(x) = \sum_{y=1}^{w+1} (y)\ p(y\text{-}1)$, by setting x+1 = y

$$= \sum_{y=1}^{w+1} y \text{ x probability (age at death is y)} = \text{mean (expected) age at death} .$$

Hence, $A_0 = \dfrac{N}{\text{Mean age at death}}$

This completes the proof of the theorem.

1.15.2 GROUP REPLACEMENT POLICY OF ITEMS THAT FAIL SUDDENLY AND COMPLETELY

The rate of replacement and total cost involved in group replacement is based on the following theorem.

Theorem:

(a) *"Group replacement should be made at the end of t^{th} period if the cost of individual replacements for the period 't' is greater than the average cost per period through the end of the period, 't'".*

(b) *"Group replacement is not advisable at the end of period 't', if the cost of individual replacement at the end of period 't–1' is less than the average cost per period through the end of t^{th} period".*

Proof:

Let, N = Total number of items in the system

N_t = Number of items failing during time 't'

C(t) = Total cost of group replacement after time period 't'

C_1= Individual replacement cost on failure

C_2= Per item cost of replacement in a group

Then, clearly, $C(t) = C_1[N_1 + N_2 + N_3 + ... + N_{t-1}] + C_2 N$

Therefore, the average cost of group replacement per unit period of time during a period 't' will be

$$F(t) = \frac{C(t)}{t} = \frac{C_1[N_1 + N_2 + N_3 + ...+ N_{t-1}] + C_2 N}{t} \qquad \text{---(1.25)}$$

Now, in order to determine the replacement age 't', the average cost per unit period should be minimum. The condition for minimum of F(t) is ,

$$\Delta F(t-1) < 0 < \Delta F(t) \qquad \text{---(1.26)}$$

Now, $\Delta F(t) = F(t+1) - F(t) = \frac{C(t+1)}{(t+1)} - \frac{C(t)}{(t)} = \frac{C(t)+C_1 N_t}{(t+1)} - \frac{C(t)}{(t)}$

$$\frac{t\,C_1 N_t - C(t)}{t\,(t+1)} = \frac{C_1 N_t - C(t)/t}{t\,(t+1)} \qquad \text{---(1.27)}$$

which must be greater than zero for minimum F(t). That is $C_1 N_t > C(t)/t$ ---(1.28)

Similarly, from $\Delta F(t-1) < 0$, we get $C_1 N_{t-1} < \frac{C(t)}{t}$ ---(1.29)

Thus, $C_1 N_{t-1} < \frac{C(t)}{t} < C_1 N_t$.

It can also be noted that, $C_1 N_t > \frac{C(t)}{t} \Rightarrow \frac{C_1 N_t}{t+1} > \frac{C(t)}{t\,(t+1)}$

$$\Rightarrow \frac{C_1 N_t}{t+1} + \frac{C(t)}{(t+1)} > \frac{C(t)}{t\,(t+1)} + \frac{C(t)}{(t+1)} \Rightarrow \frac{C(t) + C_1 N_t}{t+1} > \frac{C(t)}{(t+1)}\left[1 + \frac{1}{t}\right] \Rightarrow \frac{C(t+1)}{t+1} > \frac{C(t)}{(t)}$$

Or, the average cost for (t+1) periods > average cost for 't' periods.

Table 1.2 shows the failure rates of items for k periods.

Table 1.2 Failure rates of items

Period	:	1	2	3	4	5	….	K
Percent failing by the end of period	:	λ_1	λ_2	λ_3	λ_4	λ_5	….	λ_k

Let, p_i = the probability that an item of sudden failure nature, which was new when placed in position for use, fails during the i^{th} period of its life. Thus, the following probability distribution is obtained assuming to replace the items as and when they fail.

p_1 = the probability of failure in 1^{st} period = $\lambda_1/100$

p_2 = the probability of failure in 2^{nd} period = $(\lambda_2 - \lambda_1)/100$

p_3 = the probability of failure in 3^{rd} period = $(\lambda_3 - \lambda_2)/100$

p_4 = the probability of failure in 4^{th} period = $(\lambda_4 - \lambda_3)/100$

p_5 = the probability of failure in 5^{th} period = $(\lambda_5 - \lambda_4)/100$

…………………………………………………………..

p_k = the probability of failure in k^{th} period = $(\lambda_k - \lambda_{k-1})/100$

Since the sum of probabilities can never be greater than unity, and when the sum of all the above probabilities, say up to 5^{th} period is unity, the further probabilities p_6, p_7, p_8....so on, will be zero . Thus, all items are sure to fail by the end of 5^{th} week.

Furthermore, it is assumed that (i) the items that fail during a period are replaced just before the end of that period, and (ii) the actual percentage of failures during a period for a sub-population of items with the same age is the same as the expected percentage of failure during the period for that sub-population .

Let, N_i = the number of replacements made at the end of the i^{th} period, if all the N items are new initially . Thus,

$$N_0 = N_0$$

$$N_1 = N_0 p_1$$

$$N_2 = N_0 p_2 + N_1 p_1$$

$N_3 = N_0p_3 + N_1p_2 + N_2p_1$

$N_4 = N_0p_4 + N_1p_3 + N_2p_2 + N_3p_1$

$N_5 = N_0p_5 + N_1p_4 + N_2p_3 + N_3p_2 + N_4p_1$

$N_6 = 0 + N_1p_5 + N_2p_4 + N_3p_3 + N_4p_2 + N_5p_1$

$N_7 = 0 + 0 + N_2p_5 + N_3p_4 + N_4p_3 + N_5p_2 + N_6p_1$

…….and so on up to k^{th} period.

It has been studied that expected number of items failing in each period increased initially and then decreased. Thus, the number of failures or replacements will continue to oscillate and ultimately, the system settles down to a steady state in which the proportion of items fail in each period is reciprocal of their average life .

Individual Replacement:

The mean age of items $= 1\times p_1 + 2\times p_2 + 3\times p_3 + 4\times p_4 + 5\times p_5 +$ …. + $k.P_k$ = $\sum_{i=1}^{n} i.p_i$

The number of failures in each period in steady state = $N/\sum_{i=1}^{n} i.p_i$

The cost of replacing items individually on failure = $= \left[(N/\sum_{i-1}^{n}(i.p_i) \right] C_1$

Group Replacement:

The replacement of all N items simultaneously costs C_2 per item and replacement of an individual item on failure costs C_1. The cost of replacement of all items simultaneously, is computed as shown in Table 1.3.

It would be optimal to group replace all the items after every i^{th} period, when the cost of individual replacement in the $(i+1)^{th}$ period is greater than the average cost for i^{th} period. Otherwise, the average cost will start increasing from $(i+1)^{th}$ period onwards .

Table 1.3 Cost of replacement of items

End of period	Cost of individual replacement	Total cost of group replacement (Rs.)	Average cost per period (Rs.)
1	N_1 x C_1	$GTC_1 = N$ x $C_2 + N_1$ x C_1	GTC_1 / 1
2	N_2 x C_1	$GTC_2 = TC_1 + N_2$ x C_1	GTC_2 / 2
3	N_3 x C_1	$GTC_3 = TC_2 + N_3$ x C_1	GTC_3 / 3
4	N_4 x C_1	$GTC_4 = TC_3 + N_4$ x C_1	GTC_4 / 4

Further, when the individual replacement cost after i^{th} period is less than the group replacement cost at the end of i^{th} period the individual replacement after i^{th} period is preferable .

1.16 OTHER REPLACEMENT PROBLEMS

Apart from industrial replacement problems, replacement principles are also applicable to the problems of recruitment and staff promotion. In staffing problems, with fixed total staff and fixed size of staff groups, the proportion of staff in each group determines the promotion age, unemployment situations now-a-days can be considerably improved by the possibility of expansion. In the organizations where staff frequently float away, applying probability concepts it is possible to determine number of candidates to be recruited every year so as to maintain constant workforce in the organization. Attempts were made to determine the number of staff to be recruited every year to maintain constant size of teaching staff in Engineering College where the staff are observed floating heavily, by a model using probability concepts which can also give promotion age for the new entrants.

1.17 EQUIPMENT RENEWAL PROBLEMS

The word renewal means that either to insert a new equipment in place of an old one or repair the old equipment so that the probability density function of its future lifetime will be equal to that of new equipment .

Definition of Renewal Rate: "*The probability that a renewal occurs during the small time interval (t, t+δt) is called the renewal rate at time 't', where time 't', is measured from the instant of the first machine was started. It is denoted by h(t).dt and also called the **Renewal Density Function***"**.**

The renewal rate of an equipment is asymptotically reciprocal of the mean life of the equipment i.e., $h(t) = \frac{1}{\lambda}$ = reciprocal of mean life.

Equipment renewal comes under major preventive maintenance activity which may include replacement of few parts/subsystems or conditioning the equipment. With respect to this detailed mathematical models are not discussed here as the main focus area is on blocks and block replacements.

Summary:

The concepts and importance of maintenance in production environment are discussed in this chapter. The two types of maintenance – Breakdown maintenance and Planned maintenance are explained in detail. The objective of maintenance work should be to strike a balance between the availability and the overall running costs. The possibilities of replacement of the equipment to ensure its normal performance are also discussed. This chapter also discusses two categories of replacement techniques for determining the best replacement strategies for the items that deteriorate with time and those do not deteriorate but fail suddenly. These models are discussed with respect to the parameters like maintenance cost, and time value of money.